UCRL-TR-221785

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Heat Loss by Helicity Injection II

T.K. Fowler

June 2, 2006

This work was performed under the auspices of the U.S. Department of Energy by University of California, Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

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Abstract

Arguments are reviewed showing that helicity transport always flattens the temperature profile, yielding unit current amplification in SSPX and flat temperature profiles in RFP's whenever the dynamo is active. The argument is based on transport theory yielding a hyper-resistivity $\Lambda \approx (c^2/\omega_{pe}^2)\, D_M$ and $D_M = \alpha\chi_e$. Earlier theoretical work assuming a drifting Maxwellian electron distribution found $\alpha \approx 1$, but even a large α (representing distortions of the distribution) would not change the qualitative conclusion that hyper-resistive transport flattens the temperature profile as long as $(\Lambda/a^2\chi_e) << 1$. The theory could be tested by deriving Λ from helicity transport in SSPX, by analogy with recent analysis yielding χ_e from heat transport. If the predicted small ratio $(\Lambda/a^2\chi_e)$ is confirmed, efforts to increase current amplification in SSPX must be based on scenarios consistent with slow helicity transport compared to heat transport (pulsed reactor, multipulse, neutral beam injection).

1. Introduction

The revival of interest in spheromaks that led to SSPX was motivated by high temperatures achieved in CTX [1]. This note is an update of Ref. [2], written in 1994, in which I tried to account for the high temperatures in CTX and, from this, to obtain a buildup scenario yielding high current amplification. It was noted that CTX temperatures could be explained by S-scaling [3] applied to the Rechester-Rosenbluth thermal diffusivity χ_{RR} in a tangled magnetic field [4], giving:

$$\partial(nT)/\partial t - \nabla\cdot n\chi_e\nabla T \quad = \quad \eta j^2 \quad = \quad (B^2/2\tau\mu_o) \qquad (1)$$

where we used $\mu_o\, j = \lambda B$ giving the ohmic decay time $\tau = (\mu_o /2\eta\lambda^2)$. For steady state with $\chi_e = \chi_{RR}$, we obtain:

$$\chi_{RR} = v_e L_C(\delta B^2/B^2) \quad (2)$$

$$\beta = 2\mu_o nT/B^2 = (v_A/v_e)[1/(\delta B^2/B^2)S] \quad (3)$$

In Eqs. (2) and (3), $\delta\mathbf{B}$ is the magnetic fluctuation relative to a mean field $\mathbf{B}$; $L_C \approx a$ is the correlation length with minor radius a; v_e is the electron thermal speed; and $S = v_A\tau/a$ with Alfven speed v_A .

For S scaling ($\delta B^2/B^2 = S^{-1}$), Eq. (3) gives $\beta = v_A/v_e \approx (m_e/m_i)^{1/3}$, which fit CTX results for two very different values of B and T. By similar arguments Connor and Taylor had predicted constant beta in RFP's [5]; and this prediction of constant $\beta \approx 5$ - 10 % seemed to be even better confirmed in SSPX. This seeming success in predicting temperatures led us to conclude, incorrectly, that even weak S-scaling fluctuations must transport helicity fast enough to exceed resistive losses, thereby building up the current. It was through NIMROD simulations that we came to realize that high temperatures were achieved only if helicity transport ceases. Then flux surfaces close, giving roughly ion classical transport that yields a similar $\beta \approx (m_e/m_i)^{1/4}$ [5], now thought to be the more likely explanation for high temperatures.

Here we show that helicity transport is always slower than heat transport. Then helicity transport across any width Δ should always flatten the temperature, giving the same result as if field lines connect across Δ, even if flux surfaces close intermittently.

In Section 2, we review helicity transport theory, giving flat temperature profiles in Section 3, which in turn accounts for low current amplification in Section 4. In Section 5 we suggest experimental tests, by extracting helicity transport coefficients from SSPX data much as the heat transport coefficient χ is extracted now. Section 6 discusses results in the context of buildup scenarios that might yet succeed in achieving high current amplification.

2. Helicity Transport Theory

Helicity transport is described by the hyper-resistive Ohm's Law given by:

$$\begin{aligned} E_{||} &= (m/ne^2)\{\nu j_{||} - \nabla_\perp\cdot D_M \nabla_\perp j_{||}\} \\ &= \eta j_{||} - B^{-1}\nabla_\perp\cdot B^2 \Lambda\nabla_\perp\lambda \end{aligned} \quad (4)$$

$$\Lambda \quad = \quad (m/\mu_o ne^2)D_M \quad = \quad (c^2/\omega_{pe}^2)D_M \qquad (5)$$

In the second line of Eq. (4) we have put results in helicity-conserving form [6] with $\lambda = \mu_o j_{||}/B$, though as yet we cannot justify moving n inside the derivative.

Eq. (4) is essentially the moment equation for electron momentum neglecting inertia, with collision frequency ν giving resistivity $\eta = (m/ne^2)\nu = \mu_o(c^2/\omega_{pe}^2)\nu$. Any kind of turbulence produces an additional effective collision frequency $\nabla_\perp \cdot D_M \nabla_\perp$ with momentum diffusion coefficient D_M.

Equivalents to Eq. (4) have been derived in various ways [6,7,8,9]. The most fundamental derivations use Kaufman's transport theory in action-angle space [10], by which any process that transports electron momentum by Eq. (4) makes a corresponding contribution to the electron heat diffusivity, giving:

$$D_M \quad = \quad \alpha\chi_e \qquad (6)$$

Following Kaufman and Mahajan, Hazeltine and Hitchcock [11], in his Ph. D. thesis Gatto [12] found $\alpha \approx 1$ when he assumes that the averaged distribution f_o retains the form of a drifting Maxwellian, perhaps valid here but not valid, for example, if $E_{||}$ were strong enough to produce a runaway electron beam to carry the current. Also, Kaufman's theory should include MHD hyper-resistivity [13], and therefore Gatto's relationship between hyper-resistivity and heat transport should apply also to MHD, though there remains some uncertainty whether MHD is additive to the "kinetic" results in Refs. [7] and [8]. These points are being reinvestigated in a paper I am writing with Gatto, soon to be submitted for publication. Meanwhile, we take $\alpha \equiv D_M/\chi_e$ representing distortions of the distribution away from a drifting Maxwellian as a fitting parameter to be determined by experiment in Section 5.

3. Flattening of the Temperature Profile

If instability is strong enough to cause helicity to be transported, the right hand side of Eq. (4) must be negative so that helicity flow exceeds resistive losses. By inspection, making the right hand side of Eq. (4) negative requires:

$$\alpha\chi_e \quad = \quad D_M \quad > \quad \nu\Delta^2 \quad = \quad (\omega_{pe}{}^2/\lambda^2c^2)(\Delta^2/\tau) \qquad (7)$$

where we use Eq. (6), and $\nabla \approx 1/\Delta$. Introducing Eq. (7) into Eq. (1) for steady state and dividing by $n\chi_eT$ gives:

$$-\nabla\cdot n\chi_e \nabla T \quad > \quad n(\omega_{pe}{}^2/\lambda^2c^2)(\Delta^2/\alpha\tau)\nabla^2T \quad = \quad (B^2/\tau\mu_o) \qquad (8)$$

$$\delta T/T \quad \equiv \quad \alpha(\Delta^2\,\nabla^2T/T) \quad < \quad \alpha[1/\beta(\omega_{pe}{}^2/\lambda^2c^2)] \qquad (9)$$

Here δT is the temperature change across the scalelength Δ for local gradients in $j_{||}$ in Eq. (4). For typical parameters, $\delta T/T << 1$. It is this that accounts for flat temperature profiles in MST whenever the dynamo is active, and for limitations on current amplification in SSPX.

Note that, for magnetic turbulence, this result does not depend explicitly on $\delta B/B$; instead, instability forces $\delta B/B$ to satisfy Eq. (7). Finding the actual magnitude of δB requires a self-consistent calculation of the turbulence spectrum and the toroidally-averaged current density.

4. Flux and Current Amplification in Spheromaks

For the spheromak, Eq. (9) implies that during gun injection the average electron temperature T is equal to that in the flux core connected to the gun, namely, that for short open field lines [3]:

$$T \quad \approx \quad 0.4(V - V_S) \qquad (10)$$

with gun voltage V and sheath voltage V_S. Thus, during buildup, we should use Eq. (10) in calculating the resistivity appearing in the Ohm's Law, Eq. (4).

To calculate helicity buildup, we dot Eq. (4) into **B** and integrate over any volume including the flux core:

$$dK/dt \equiv d/dt \int \mathbf{A}\cdot\mathbf{B} = 2V\Phi - \int d\mathbf{S}\cdot\boldsymbol{\Gamma} - K/\tau \tag{11}$$

$$\boldsymbol{\Gamma} = -2B^2\Lambda\nabla\lambda \tag{12}$$

where Φ is the gun bias flux and $\int d\mathbf{S}\cdot\boldsymbol{\Gamma}$ is helicity flow across the surface, usually taken zero if the volume includes the entire flux conserver. Dropping the surface term gives at any time during the buildup [14]:

$$K \approx \psi_{POL}\psi_{TOR} \approx \psi_{POL}^2 \leq 2V\Phi\tau \tag{13}$$

$$\psi_{POL}/\Phi \leq (2V\tau/\Phi)^{1/2} \tag{14}$$

Eq. (14) gives the flux amplification (ψ_{POL}/Φ) with τ calculated for resistivity η with T in Eq. (10). For large gun current [14], we can neglect the impedance that Eq. (11) presents to the gun and obtain the current amplification by substituting $V \geq I_{GUN} R_{\Omega}$ into Eq. (14), where $R_{\Omega} = \eta_C L B_C/\Phi$ is the resistance of the flux core with length L and area Φ/B_C where B_C is the poloidal field at the geometric axis. Using $\psi_{POL} = 1/2\mu_o I_{TOR} a$ and $\tau = (\mu_o /2\eta\lambda^2)$ with $\lambda a \approx 2$, the result is [14]:

$$I_{TOR}/I_{GUN} \leq (B_C L/\mu_o I_{TOR}) \tag{15}$$

where, consistent with the flattened temperature profile, we took η inside the separatrix to equal η_C in the flux core. In the limit of large I_{TOR}, $B_C \propto I_{TOR}/a$ whereby the maximum current amplification I_{TOR}/I_{GUN} approaches a constant $\propto L/a$ characteristic of the flux conserver, with a value of order unity in agreement with SSPX results [14].

Eqs. (14) and (15) can be derived from modified Taylor relaxation theory using minimization of the energy dissipation rather than minimizing the energy itself [15].

5. Experimental Tests of the Transport Theory

The most direct test of the theory would be obtained by substituting Eq. (6) into Eqs. (1) and (4) giving T and λ profiles to be compared with experimental data. Lacking a reliable calculation of D_M, one could simply take D_M to be a constant in Eqs. (4) and (5), to be determined by fitting λ profile data, as was done in Refs. [3] and [16] using the Ohm's Law of Eq. (4) to calculate the λ profile using the Corsica code. The new feature would be a simultaneous T profile obtained by including Eq. (1) in Corsica in addition to the hyper-resistive Ohm's Law. One should also include a second fitting parameter $\alpha = D_M / \chi_e$, as discussed in Section 2.

Indirect tests of the transport theory are provided by comparison of Eq. (10) with temperatures measured during helicity injection, and the flux and current amplification derived from Eq. (10) in Section 4.

Finally, NIMROD simulations have shown that spikes on the gun voltage represent events converting toroidal flux to poloidal flux, an almost-instantaneous helicity transport event [17]. Thus one might obtain direct evidence for maximum helicity transport rates and for temperature flattening during voltage spikes.

A voltage spike δV is found by perturbing Eq. (11) applied to a volume including only the flux core with surface area A_C. If we ignore helicity buildup and loss inside the flux core, this gives:

$$2\Phi\delta V \approx \delta\int d\mathbf{S}\cdot\mathbf{\Gamma} = (2A_C B^2 \nabla\lambda)\delta\Lambda \tag{16}$$

Here we neglected $\delta\nabla\lambda$ near the flux core. Dividing Eq. (16) by the steady state $2\langle V\rangle\Phi \approx (2A_C B^2\nabla\lambda)\langle\Lambda\rangle$ gives:

$$\delta\Lambda/\langle\Lambda\rangle \approx \delta V/\langle V\rangle \tag{17}$$

Thus voltage spikes are indicative of the maximum Λ near the flux core, as compared with the time-space average $\langle\Lambda\rangle$ obtained from Corsica fits.

A jump in Λ during a voltage spike should produce a corresponding jump in χ_e, most easily observed as a temperature flattening event inside a local region of intermittently closed flux where low turbulence allows T to grow beyond the limits of Eq. (10), as is observed in some NIMROD runs [17]. According to NIMROD, this usually occurs near the magnetic axis.

Instantaneous measurements of temperature near the magnetic axis before and after a voltage spike, using the double-pulsed Thomson capability soon to be available, might be able to observe a high local temperature during a quiet time and subsequent collapse of the temperature during a spurt of helicity injection correlated with a voltage spike [18].

6. Conclusions

We have concluded from transport theory that helicity transport across any region flattens the electron temperature profile in that region, and it is this that limits current buildup in SSPX.

We have suggested ways that the transport theory could be tested. Indirect evidence based on limits on current amplification suggests that the theory is basically correct, and the discussion in the Appendix shows that the theory applies to any process that might transport helicity diffusively, not only tearing.

If further analysis continues to support the theory, efforts to increase current amplification in SSPX must be based on scenarios consistent with slow helicity transport. Three scenarios that meet this requirement are the pulsed reactor, multipulsing and current drive by neutral beams. There may be others.

The pulsed reactor could work because buildup is accomplished without current amplification [19]. The main issues are how much magnetic energy is lost in a transition from the Taylor state produced by electrostatic injection to a stable mode of decay, and whether a stable mode exists that is not supported by gun current at the edge. The latter point has gained support by Pearlstein's results showing equilibria with zero λ at the edge that are stable to tearing in the straight-cylinder approximation [20].

Multipulsing, already explored to some extent [3], might have a better chance by reducing the bias flux and gun current after the initial formation phase [21], as is currently being explored on NIMROD [22]. Then helicity would be injected in a succession of small pulses. Again current amplification is not required for a single pulse. Success requires, first, that injection does not break flux surfaces in the interior, and secondly, that flux closure detaches each pulse from the gun, thereby allowing the newly injected helicity to merge into the spheromak already present [21]. Merger is aided by mutual attraction between toroidal current in a new pulse and that in the spheromak. Flux closure could be accomplished either inside the gun as the pulse leaves the gun, or in the flux conserver itself.

Thirdly, the possibility of a stable state with no gun current suggests that other methods of current drive such as neutral beams could build up and maintain this state, perhaps starting from a "target" produced by gun injection [20].

That helicity transport flattens ∇T has been suspected for many years [9, 22], but the theory is complicated in detail and only now are experiments and NIMROD simulations sufficiently advanced to warrant the kind of experimental campaign needed to test the theory in detail.

Acknowledgement

I would like to thank Don Pearlstein and Bick Hooper for numerous and ongoing discussions of the topic of this paper, and Renato Gatto whose Ph.D. thesis is quoted in Ref. [12].

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